

# Virtual reality for animal navigation with camera-based optical flow tracking

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## Abstract

*Background* Virtual reality combined with spherical treadmills is used across species for studying neural circuits underlying navigation.

*New Method* We developed an optical flow-based method for tracking treadmill ball motion in real-time using a single high-resolution camera.

*Results* Tracking accuracy and timing were determined using calibration data. Ball tracking was performed at 500 Hz and integrated with an open source game engine for virtual reality projection. The projection was updated at 120 Hz with a latency with respect to ball motion of  $30 \pm 8$  ms.

*Comparison with Existing Method(s)* Optical flow based tracking of treadmill motion is typically achieved using optical mice. The camera-based optical flow tracking system developed here is based on off-the-shelf components and offers control over the image acquisition and processing parameters. This results in flexibility with respect to tracking conditions - such as ball surface texture, lighting conditions, or ball size - as well as camera alignment and calibration.

*Conclusions* A fast system for rotational ball motion tracking suitable for virtual reality animal behavior across different scales was developed and characterized.

*Keywords:* Virtual Reality, Navigation, Optical Flow, Spherical Treadmill, Ball Tracking, Drosophila, Real-Time Image Processing

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# 1. Introduction

Virtual reality (VR) is used across species for studying neural circuits underlying behavior [1]. In many implementations, animals navigate through virtual realities on a spherical treadmill - a ball which can be freely rotated around its center of mass [1, 2, 3, 4, 5].

Tracking of ball rotation is typically accomplished using optical mice which are based on low-resolution, high-speed cameras integrated with a light source for measuring displacements when moving across a surface. Movement across the surface results in optical flow - the displacement of features across the camera sensor. Such features can for example be speckle-like reflections from surface roughness; comparing these speckle images between different frames then allows computing the displacement using hardware-integrated image processing.

Optical mice come however with some limitations for measuring ball rotation. First, a single optical mouse measures displacements only in two directions and therefore two mice are required for tracking all three degrees of freedom of ball motion. Secondly, limited or no control over the onboard processing algorithms as well as camera settings requires careful calibration. In particular, if the mouse sensors can't be placed in direct proximity of the ball surface, accurate alignment of the two sensors as well as calibration with respect to surface properties and lighting conditions is necessary [5]. As an approach that overcomes some of these limitations, real-time tracking was developed with a single high-resolution camera for situations where a uniquely patterned ball can be used [6]. In that case, ball orientation was calculated by matching each recorded frame to a map of the entire ball surface pattern. Using a high-resolution cameras allows control over all recording parameters and offers the freedom to choose a custom algorithms and test its performance with simulated data [6].

Here, we combine the flexibility of optical flow-based tracking with the advantages of using a high-resolution camera. The developed system tracks optical flow of rotational ball movement at 500 Hz using a single camera and is integrated with an open source virtual reality environment and two projectors [7] with a refresh rate of 120 Hz [8] and overall latency of  $30 \pm 8$  ms - from detecting ball movement to projecting the updated virtual reality image. This is achieved using off-the-shelf hardware components and open source software. The system is easy to align and calibrate, can be used under different conditions and at different scales, and can be integrated with

38 two-photon imaging or electrophysiology.

## 39 **2. Materials and Methods**

### 40 *2.1. Virtual reality setup components*

41 A schematic of the setup is shown in Figure 1. Two digital micromirror  
42 devices (DMD, DLP LightCrafter 6500 by Texas Instruments) were used for  
43 projecting the VR onto an angled screen from two sides [8, 7]. The projectors  
44 are FullHD digital micromirror devices with HDMI and DisplayPort inter-  
45 faces allowing them to display images the same way as standard monitors.  
46 In our setup, the DMD was set to use DisplayPort input and display the  
47 frames at a maximum frame rate of 120 Hz (see Appendix for details). This  
48 frame rate can only be achieved when frames are displayed with bit depth  
49 1 (binary images). To display different gray levels, a dithering technique is  
50 used and the density of white pixels is varied proportionally to the brightness  
51 level of the displayed region with an ordered Bayer matrix [9] (see Appendix  
52 for details). Two collimated blue LEDs (M470L3, Thorlabs) were used as  
53 light sources. Light reflected off the DMD was projected from the outside  
54 (seen from the fly’s perspective) onto a screen made from black paper.

55 The spherical treadmill used for the current experiments is described in  
56 [5]. Briefly, a 6 mm diameter polyurethane foam ball is held in a cup-shaped  
57 holder and suspended in an air stream. The ball is illuminated with two  
58 IR LEDs and monitored with a camera. A *Basler acA640-750um* camera  
59 was used for tracking. The camera uses a USB 3 interface to communicate  
60 with the PC and the bandwidth of 350 MB/s allows transfer of full-frame  
61 images ( $640 \times 480 \times 8\text{bpp}$  mono color) at a rate of 751 frames per second  
62 (fps). For tracking, a frame rate of 500 fps was used, with an exposure  
63 time of 100 microseconds under infrared illumination. The resolution was  
64  $224 \times 140$  pixels which results from cropping and  $2 \times 2$  binning of the full  
65 frame. Binning (summing the signal from adjacent pixels into a single one)  
66 increased signal-to-noise ratio and reduced the image readout and transfer  
67 time. A Computar camera objective M2514-MP2 F1.4,  $f = 25\text{mm}$  together  
68 with a Computar EX2C extender were used for imaging the ball [5].

### 69 *2.2. Integration with virtual reality*

70 For virtual reality applications a 3D rendering game engine (Urho3D, an  
71 open-source C++ game engine [10]) was used and the x, y- and z-rotations of  
72 the ball were mapped to planar motion (forward-, sidestepping- and turning

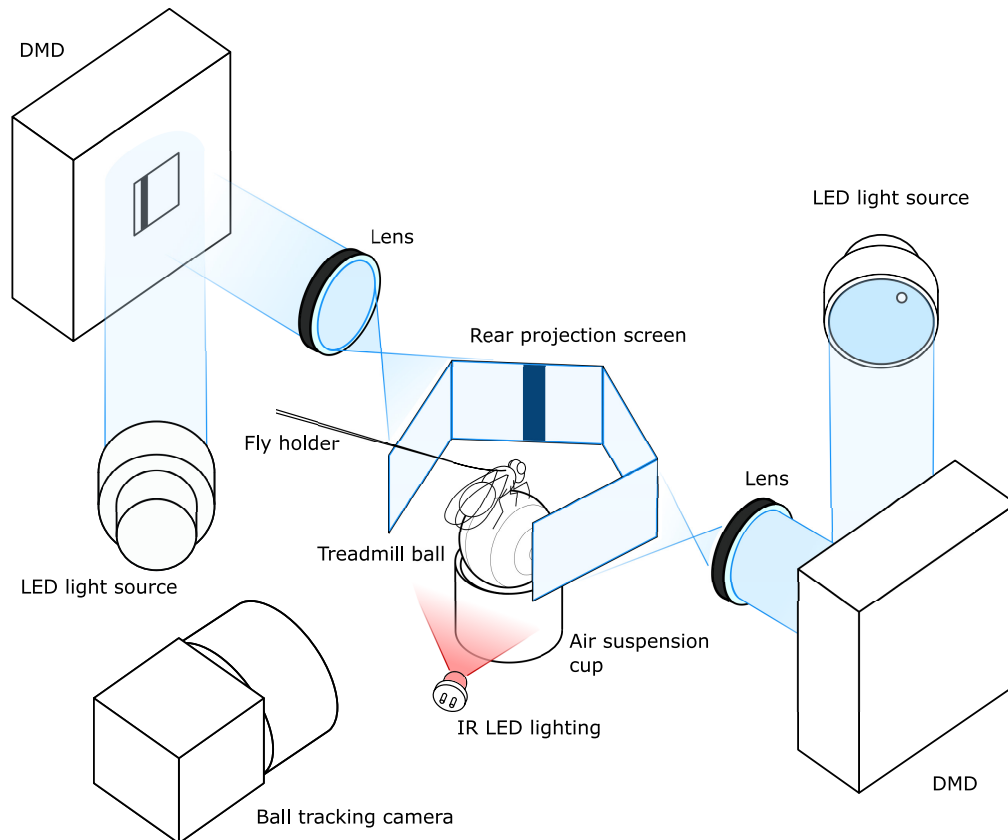


Figure 1: Schematic of behavior setup for *Drosophila melanogaster*. A 6-mm polyurethane ball is air-suspended in a holder and serves as an omnidirectional treadmill. The fly is glued to a thin metal wire. A rear projection screen is used for displaying the virtual reality (VR) in the fly's field of view. Two DMDs, LED light sources, and two projection lenses are combined to project from two sides onto the screen. A single tracking camera is set up to capture ball images at the rate of 500 fps. Infrared LED lighting invisible to the fly is used for illuminating the ball.

73 components) of the animal in the virtual reality (this was done similarly to  
74 [6], see Appendix for details).

75 The application is designed to run ball tracking and environment rendering  
76 in two separate execution threads. The image processing thread is a loop  
77 acquiring camera frames and calculating ball displacements, tuned to run  
78 one iteration in under 2 ms, resulting in a tracking frequency of 500 fps. The  
79 rendering thread runs the game engine, which models a virtual environment  
80 in 3D.

81 Setting up the virtual environment requires specifying the arena geometry  
82 as well as lighting and rendering properties. Further, the virtual cameras  
83 have to be configured such that the virtual environment is rendered from the  
84 angles corresponding to the position of the screen. Additional parameters are  
85 required to describe the mapping of ball rotations to movement in the virtual  
86 environment; these transformations take into account the size of the ball and  
87 the orientation of the tracking camera relative to the forward direction of  
88 the animal. This is all done with the in-built Urho3D engine's scripting  
89 language which also describes each environment in a separate script file,  
90 containing instructions about which objects are to be created in the scene.  
91 In each update of the game engine the ball tracking signal is interpreted as the  
92 displacement of the animal, and the virtual camera positions and orientations  
93 are updated before being rendered and displayed on the screen.

94 Other game engine capabilities used in the application are dithering shader  
95 to output binary frames suitable for the DMD and networking to broadcast  
96 the state of the VR to any client, for example for monitoring the VR or a  
97 script that triggers any other external stimuli. The application can also be  
98 used without visual output for tracking the animal's activity. The configu-  
99 ration of the virtual environment is described in detail in the Appendix.

100 The tracking data is saved in a text log file: for each camera frame a new  
101 line is added with the timestamp of the frame, x-, y-, z- ball displacements  
102 and x-, y-, z- position of the animal in the virtual environment. Additionally,  
103 the tracking camera is set to output a frame trigger pulse with each recorded  
104 frame, which can be used for synchronization with other applications, such  
105 as two-photon imaging.

106 All experiments were performed on a PC with an Intel Xeon E5-1620 v3  
107 @ 3.50GHz (8 cores) CPU, 32Gb DDR 4 @ 2993MHz of RAM, a NVIDIA  
108 Quadro K620, 2Gb DDR3 RAM, 384 CUDA cores GPU and Microsoft Win-  
109 dows 8.1 Enterprise edition operating system. OpenCV 3.4.1 and opencv-  
110 contrib module were compiled using Microsoft Visual Studio Toolkit 14.0

(VS2015) in release configuration with CUDA 9.1 for tests of the GPU-accelerated algorithms.

### 3. Theory/calculation

#### 3.1. Modeling of optical flow camera projections

In this section a model is developed that describes how optical flow generated by ball rotations around different axes is projected onto the camera sensor. Predictions of this model will be used as fit functions to extract ball rotation parameters from measured optical flow distributions.

Ball motion is described by specifying an axis of rotation and an angular velocity (axis-angle representation [11]). Assuming a Cartesian coordinate system  $O$  with the origin at the center of the ball and a corresponding ISO-conventional [12] spherical coordinate system  $S$  as shown in Figure 2, a point  $p$  on the surface of the ball can be described with a vector  $\mathbf{p}_S = (r_{ball}, \theta_p, \varphi_p)$ . If  $O$  is selected so that its z-axis coincides with the axis of ball rotation, it becomes a convenient parametrization since only the azimuthal angle is varying with time if the ball rotates around the polar axis with angular velocity  $\omega$ :

$$\mathbf{p}_S(t) = \begin{bmatrix} r_{ball} \\ \theta_p \\ \varphi_p + \omega t \end{bmatrix}. \quad (1)$$

In the Cartesian coordinate system  $O$  the same trajectory is expressed as

$$\mathbf{p}_O(t) = \begin{bmatrix} r_{ball} \sin(\theta_p) \cos(\varphi_p + \omega t) \\ r_{ball} \sin(\theta_p) \sin(\varphi_p + \omega t) \\ r_{ball} \cos(\theta_p) \end{bmatrix}, \quad (2)$$

where  $\theta_p$  and  $\varphi_p$  are the polar and azimuth angle of a point on the surface of the ball and  $r_{ball}$  is the constant radius.

Using a pinhole camera model [13] we can express the projection of points on the ball surface onto the camera sensor. We here follow the same naming and frame orientation convention as the OpenCV library [14, 15], shown in Figure 3. The trajectory of a point on the ball surface in the camera frame  $C$  is

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = R \begin{bmatrix} r_{ball} \sin(\theta_p) \cos(\varphi_p + \omega t) \\ r_{ball} \sin(\theta_p) \sin(\varphi_p + \omega t) \\ r_{ball} \cos(\theta_p) \end{bmatrix} + T, \quad (3)$$

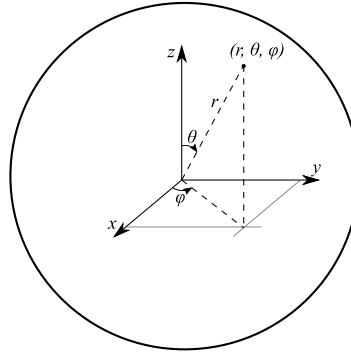


Figure 2: Spherical coordinate system complying with the ISO 31-11 convention: a point is addressed by radius  $r$ , polar angle  $\theta$  and azimuthal angle  $\varphi$

where  $R$  is the rotation matrix representing the orientation of the ball's axis of rotation and  $T$  is the position of the ball's reference frame origin in the camera frame  $C$ . The image plane coordinates  $(u, v)$  of the projected point  $p'$  are expressed as

$$\begin{aligned} u &= f_x \frac{x_c}{z_c} + center_x \\ v &= f_y \frac{y_c}{z_c} + center_y \end{aligned} \quad (4)$$

where  $f_x$  and  $f_y$  are the focal lengths expressed in pixel units and  $(center_x, center_y)$  is the image center. The ball is centered in the camera's field of view and therefore  $T = [0, 0, d]$ , where  $d$  is the distance between the center of the ball and the camera aperture.

By calculating the ball point projection between two consecutive frames at time  $t$  and  $t + \Delta t$ , with  $\Delta t$  equal to the frame period, one can calculate the displacement of the point projection and predict optical flow. If all camera parameters as well as ball location and ball size are known, the model can be used to calculate the optical flow distribution on the camera. The model was implemented using the symbolic math library SymPy.

The angular velocity vector can be expressed as a sum of three orthogonal angular velocity components. For a camera-centered ball, it is convenient to choose the axes of a frame aligned with the camera frame with its origin at the ball's center. This gives three distinct rotations which can be registered by the camera as x-, y- and z- rotations (see Figure 3).

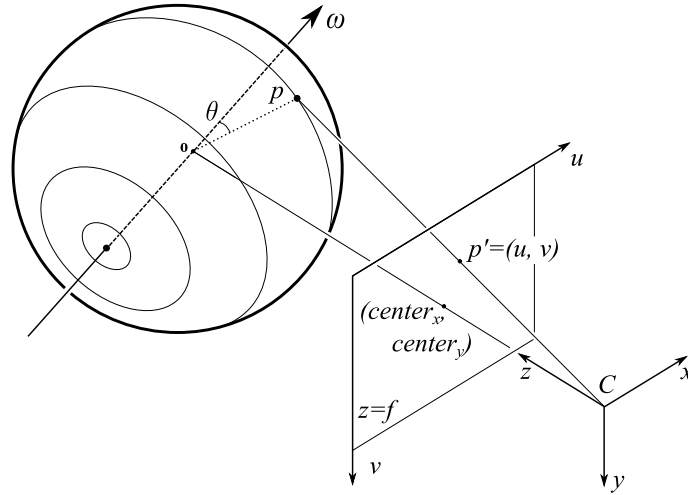


Figure 3: Projection of points on the ball with a pinhole camera model. Point  $p$  on the ball surface can be described with coordinates  $(r, \theta, \varphi)$  in a polar coordinate system aligned with the ball's instant axis of rotation. According to expression (3) it can be expressed in Cartesian coordinates  $(x, y, z)$  in the camera's reference frame  $C$  and projected onto the point  $p' = (u, v)$  in the image plane according to equation (4).

155 Filling in the matrix  $R$  in expression (3) to obtain ball rotations around  
 156 the  $x$ ,  $y$ , and  $z$  camera frame axes, respectively, and combining it with (4)  
 157 yields the trajectories of the ball's point projections on the camera (for ro-  
 158 tation around the specified axis):

$$x : \begin{cases} u = center_x - \frac{f_x r_{ball} \cos(\theta_p)}{d + r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)} \\ v = center_y + \frac{f_y r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)}{d + r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)} \end{cases} \quad (5)$$

$$y : \begin{cases} u = center_x + \frac{f_x r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)}{d - r_{ball} \sin(\theta_p) \sin(\omega t + \varphi_p)} \\ v = center_y + \frac{f_y r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)}{d - r_{ball} \sin(\theta_p) \sin(\omega t + \varphi_p)} \end{cases} \quad (6)$$

$$z : \begin{cases} u = center_x + \frac{f_x r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)}{d + r_{ball} \cos(\theta_p)} \\ v = center_y + \frac{f_y r_{ball} \sin(\theta_p) \cos(\omega t + \varphi_p)}{d + r_{ball} \cos(\theta_p)} \end{cases} \quad (7)$$

159 As seen in Figure 4 a-c, rotations around the three different camera frame  
 160 axes produce recognizably different velocity field distributions. To distinguish  
 161 them, a ring-shaped region of interest is selected around the center of the ball  
 162 (Figure 4, d-f). For each point in the ROI, two principal directions are se-  
 163 lected (radial - orthogonal to the ROI circle at this point - and tangential



164 to the ROI circle at this point, directed counterclockwise). The optical flow  
165 vector is then projected onto these directions to find its radial and tangential  
166 components with respect to the ROI (Figure 4, g-i). These radial and tan-  
167 gential components of the optical flow build distinct distributions over the  
168 ROI, which allows to separate the motion components (Figure 4, j-l).

169 These distributions can be fitted well by the following two functions

$$\begin{cases} f_{rad}(\varphi) = c_{xy \ rad}(\omega_{xy} \cdot \Delta t) \sin(\varphi + \phi) \\ f_{tan}(\varphi) = c_{xy \ tan}(\omega_{xy} \cdot \Delta t) \cos(\varphi + \phi) + c_z(\omega_z \cdot \Delta t), \end{cases} \quad (8)$$

170 where  $c_{xy \ rad}$  and  $c_{xy \ tan}$  (px/rad) are calibration factors relating the angular  
171 displacement for angular velocity vectors lying in the xy-plane to the respec-  
172 tive optical flow (see Appendix for details). Similarly,  $c_z$  (px/rad) is a factor  
173 relating angular displacement for angular velocity vectors lying on the z-axis  
174 to optical flow;  $(\omega_{xy} \cdot \Delta t)$  and  $(\omega_z \cdot \Delta t)$  are the angular displacements about  
175 a rotational axis lying in the xy-plane or on the z-axis, respectively, and  $\phi$   
176 is the orientation of the axis of rotation in the xy-plane. Note, that radial  
177 and tangential components of optical flow induced by  $\omega_{xy}$  have different co-  
178 efficients, since the magnitudes of the respective optical flow distributions do  
179 not match, while the  $\omega_z$  rotation induces only tangential optical flow, and a  
180 single coefficient  $c_z$  is sufficient. The calibration factors subsume all setup  
181 parameters, such as camera focal length, and can be determined as described  
182 below and in the Appendix.

### 183 3.2. Finding calibration factors by function fitting

184 Calibration factors were determined experimentally by recording a se-  
185 quence of frames with two cameras pointed at the ball center and positioned  
186 orthogonally to each other (see Figure 14 in the Appendix). The z-component  
187 is a planar (that is, in the camera focal plane) motion which can be measured  
188 accurately with optical flow (as was verified using simulated as well as motor  
189 actuated ground truth data), the xy- coefficient can be found by regression  
190 between the estimates of the ball motion based on the two cameras. This is  
191 described in more detail in the Appendix.

### 192 3.3. Tracking algorithm

193 The ball tracking algorithm takes two consecutive frames of ball motion  
194 and calculates the three independent rotational displacements of the ball, i.e.  
195 the x-, y- and z-axis angular displacements, by fitting their projections onto

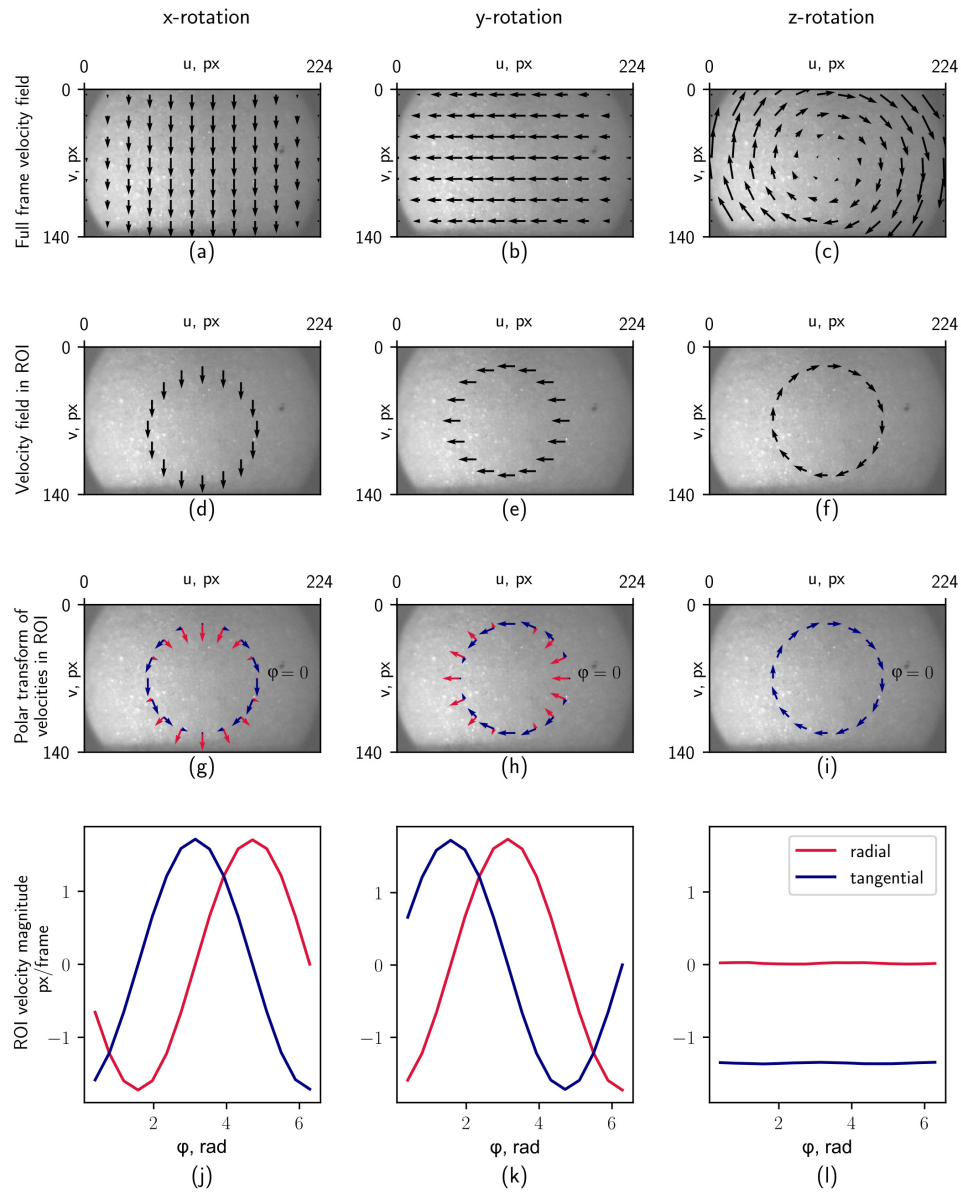


Figure 4: Predicted optical flow produced by rotations of the ball around the respective three independent axes of rotation.

the radial and tangential directions with function (8) in a circular ROI. Since the distribution of optical flow in the circular ROI is very noisy the optical flow is averaged over a band (limited by a minimal inner and maximal outer ring, see Appendix for how ROI size was determined) in the radial direction to increase robustness. This results in the following algorithm:

**Data:** Two grayscale images of the ball video  
**Result:** Angular velocity vector  $(\omega_x, \omega_y, \omega_z)$   
**Input:**  $frame_1, frame_2$

- 1 polar transform  $frame_1, frame_2$  with respect to the center of the ROI
- 2 crop the polar-transformed  $frame_1, frame_2$  to keep only the ROI
- 3  $flow_u, flow_v \leftarrow$
- 201     compute\_optical\_flow( $frame_1(cropped), frame_2(cropped)$ )
- 4  $flow_{ROI_{rad}}, flow_{ROI_{tan}} \leftarrow$  average  $flow_u, flow_v$  over radius
- 5 fit  $flow_{ROI_{rad}}, flow_{ROI_{tan}}$  with function (8) by finding  
 $(\omega_{xy} \cdot \Delta t), \phi, (\omega_z \cdot \Delta t)$
- 6  $(\omega_x, \omega_y, \omega_z) \leftarrow \omega_{xy} \sin(\phi), \omega_{xy} \cos(\phi), \omega_z$
- return**  $(\omega_x, \omega_y, \omega_z)$

Data from the algorithm's intermediate steps are shown in Figure 5. The polar transform (Figure 5, b) is calculated before computing optical flow, allowing the following optimizations: first, the ring-shaped ROI transforms into a rectangular image section, where one dimension corresponds to the azimuthal angle of the ROI  $\varphi$  and the other to the radius  $\rho$ . This means that the ROI can be isolated by cropping the image and optical flow can be calculated for a small fraction of the full frame. Secondly, in the polar-transformed ROI, the optical flow  $u$  and  $v$  components correspond to the radial and tangential motions directly, which eliminates the need of flow re-projection. The dependence of the optical flow calculation on the ROI position which results from the polar transform (see Figure 5, b) is included in the experimentally determined calibration coefficients.

Other optimizations included passing over the polar-transformed and cropped frame to the next iteration to save on those operations, and using the previous iteration's function fitting result as a prior for the solver in the next frame. Function fitting is done with Downhill Solver included in OpenCV, which implements the Nelder-Mead downhill simplex method [16].

For a compiled executable of the VR application please send an email to [ivan.vishniakou@caesar.de](mailto:ivan.vishniakou@caesar.de) or [johannes.seelig@caesar.de](mailto:johannes.seelig@caesar.de).

## 221 4. Results

### 222 4.1. Tracking algorithm implementation

223 For real-time applications the optical flow calculation was implemented  
 224 in a C++ program using OpenCV 3.4.1. Best performance in terms of pro-  
 225 cessing speed and tracking accuracy was achieved with the Farneback optical  
 226 flow algorithm on a  $60 \times 140$  ROI scaled down to  $30 \times 140$  (see the Appendix  
 227 for a comparison of a range of optical flow algorithms and for selecting ROI  
 228 parameters). Preprocessing (polar transform, cropping and resizing) together  
 229 with optical flow estimation took 1.45 ms (standard deviation  $s = 0.54$  ms),  
 230 the function fitting by downhill solver was capped at 30 iterations to keep its  
 231 run-time under 0.4 ms. These parameters allow to achieve real-time track-  
 232 ing at 500 frames per second with the current PC configuration. All time  
 233 measurements were performed with the `cv::TickMeter` class.

234 We noticed that occasionally frames were dropped, for example when  
 235 starting the program or due to other operating system related processes.  
 236 Such frame drops can be detected in the saved data file by looking for time  
 237 stamps with a separation of more than 2 ms. If a frame is dropped, the next  
 238 displacement is calculated with the latest available frame before the frame  
 239 drop.

### 240 4.2. Evaluation of tracking accuracy

The evaluation of tracking accuracy is based on comparing ground truth  $\omega_{\text{GT}}$  and estimated angular velocity  $\omega_{\text{est}}$ . Three metrics are selected for this comparison: the absolute error (in degrees per frame)

$$\varepsilon_{\text{magn\_abs}} = \text{abs}(\|\omega_{\text{est}}\| - \|\omega_{\text{GT}}\|),$$

the relative (percentage) magnitude error

$$\varepsilon_{\text{magn\_relative}} = \frac{\varepsilon_{\text{magn\_abs}}}{\|\omega_{\text{GT}}\|} \cdot 100\%,$$

and the orientation error (in degrees), which is the angle between the ground truth and estimated rotation vectors in the plane spanned by these two vectors,

$$\varepsilon_{\text{orient}} = \cos^{-1} \left( \frac{\omega_{\text{est}} \cdot \omega_{\text{GT}}}{\|\omega_{\text{est}}\| \|\omega_{\text{GT}}\|} \right).$$

241 Two ground truth datasets were generated for testing the tracking per-  
 242 formance (see Appendix for details). One consisted of simulated data which

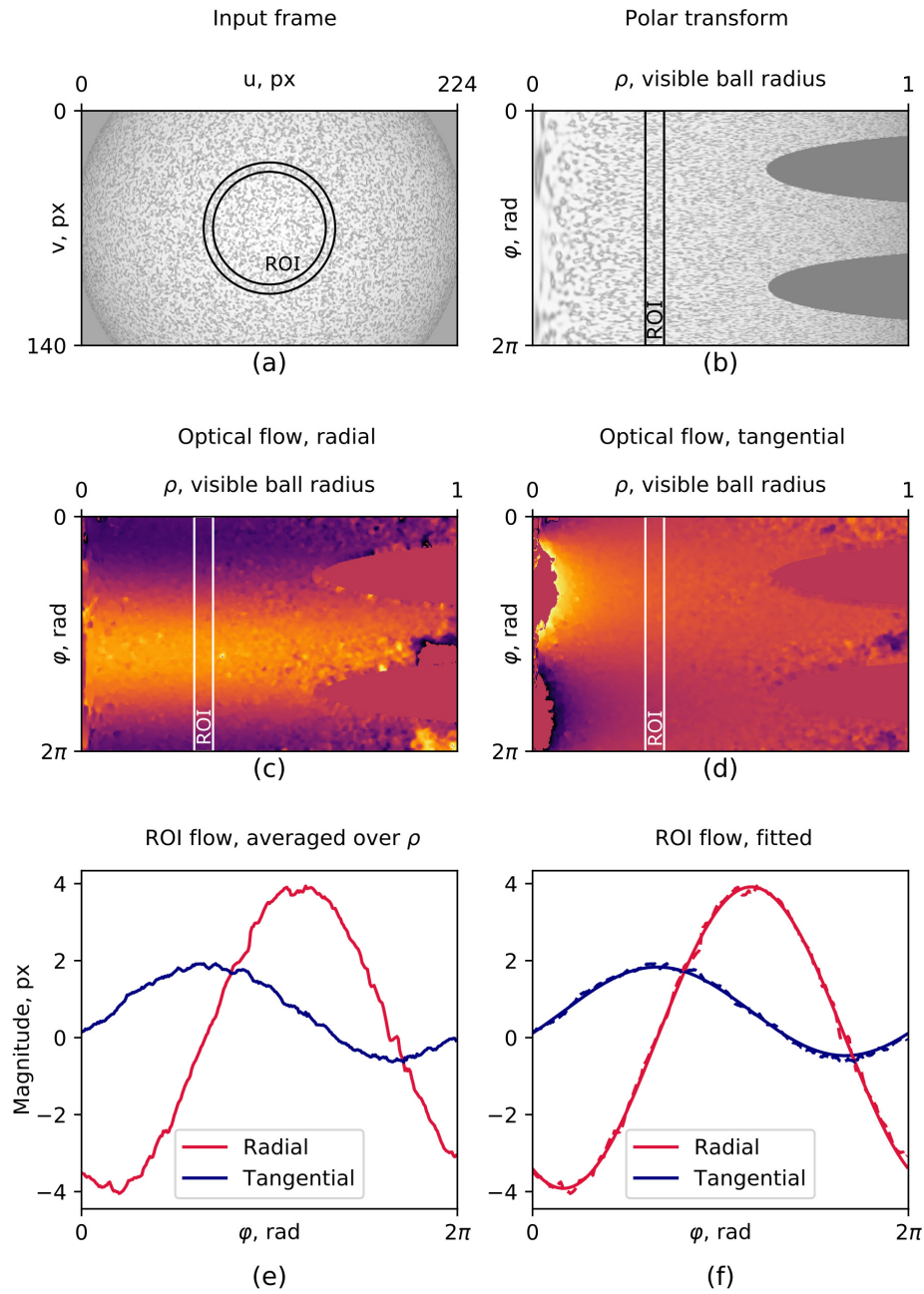


Figure 5: Ball rotation evaluation from optical flow. Input frame (a) is polar-transformed (b) and optical flow (c, d) is extracted by comparing it to the previous frame. The distribution of optical flow in the ROI (e) is then fitted with function (8) (f).

offered control of all rotation parameters, while however not fully replicating surface texture or lighting and camera conditions encountered in the actual experiment. A second dataset was generated by rotating a 60 mm ball (10× scale model for the 6 mm ball used for fly behavior) with a stepper motor.

Using the simulated data with accelerated rotations the operation range of the tracking algorithm was determined (see Appendix for details). Figure 7 shows the comparison of the integrated tracked trajectory with ground truth data. A value of 50° error for the x-axis corresponds to a 2.6 mm error in a planar trajectory. The tracking error depends on the rotation speed of the ball (for the ROI and parameters used here, see Figure 15 in the Appendix). The average tracking error stays under 10% (magnitude) and 7.5° (orientation) for rotations up to 1.70° per frame, which corresponds to 45 mm/s linear speed for a 6 mm diameter ball.

Using a stepper-motor a ball was rotated at constant speed around a single axis (see Appendix for details). The integrated rotation magnitude error and orientation errors for one rotation sequence are shown in Figure 6.

#### 4.3. VR system timing

The run-time of the tracking algorithm was measured using internal timing. The total latency of the combined tracking and VR setup was estimated using a high-speed camera filming the VR display: the display was triggered to change state, and the state change was detected with the tracking camera. This sequence contains all the latency components for one cycle of a closed loop VR experiment.

The main component of the latency is the display device input lag. It depends strongly on the selected video mode (Table 1) and the lowest values were found at 120 Hz update rate with V-sync disabled.

Table 1: Latency measurement of a camera – virtual reality – display system

Display mode	Median latency, ms
120 Hz	25 ± 8 ms
120 Hz, V-sync	33 ± 8 ms
60 Hz	50 ± 16 ms
60 Hz, V-sync	50 ± 16 ms

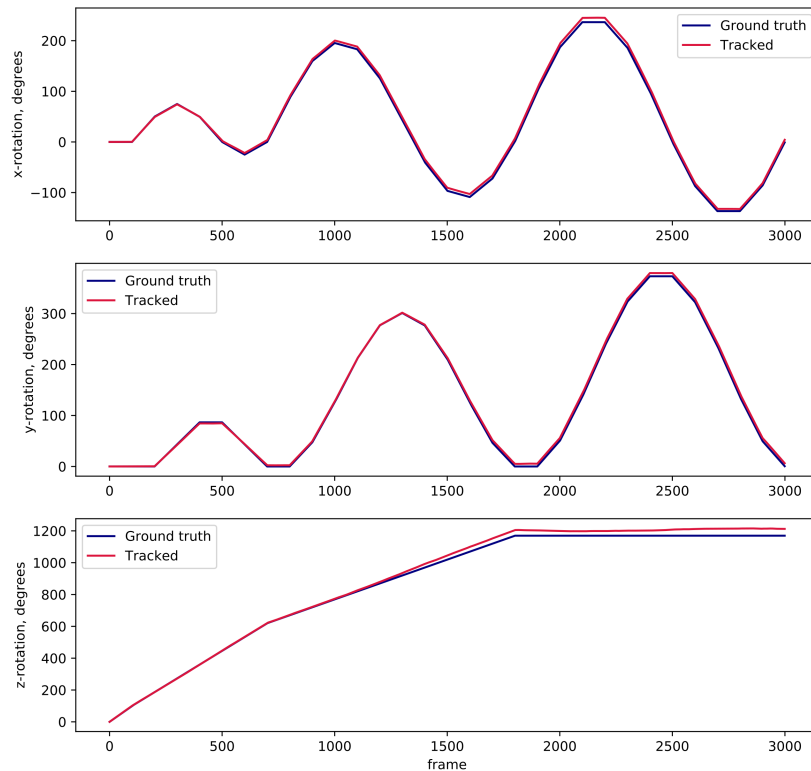


Figure 6: Integrated tracked trajectories and ground truth for stepping motor actuated ball movement at constant speed.

#### 269 4.4. Behavior experiments in VR

270 For testing the setup for fly behavior, we used a simple VR with a single  
 271 bright stripe of 10 degree width and 45 degree height projected onto a two-  
 272 part screen similar to [7] of 9 mm height and 2 times 18 mm width made out  
 273 of black paper. The fly's head was fixed to its thorax with UV glue, and the  
 274 wings were glued to the tethering pin using UV glue to encourage walking  
 275 behavior [17]. The stripe was visible for 4/5th of the flies virtual surroundings  
 276 and disappeared for 1/5th behind the fly. This resulted in robust orientation  
 277 behavior with the fly walking away from the stripe (see Figure 8).

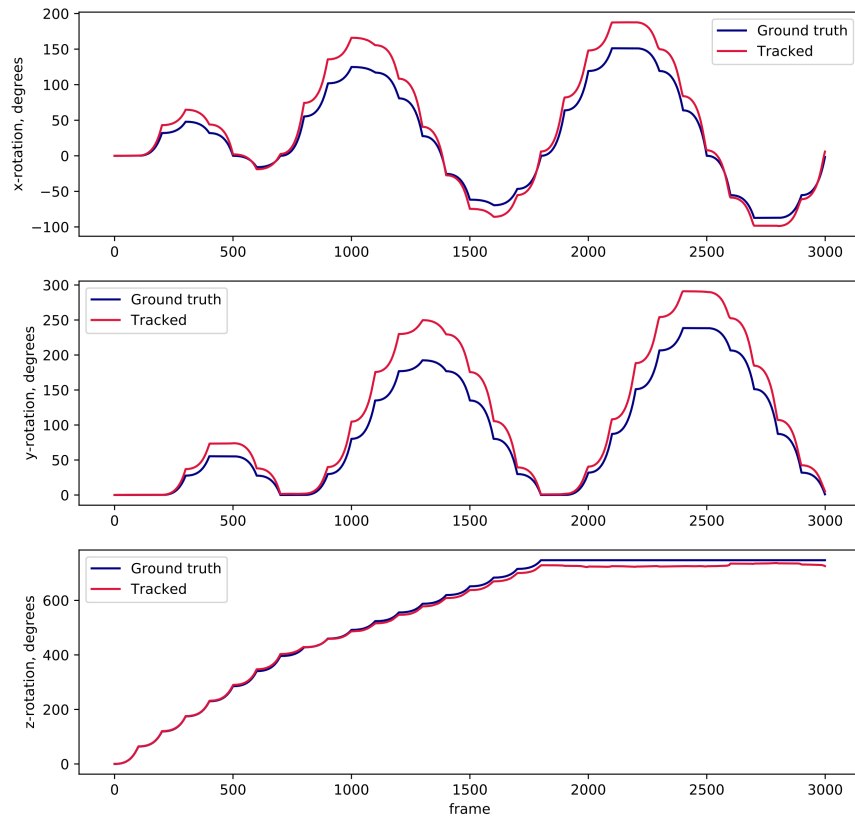


Figure 7: Integrated tracked trajectories and ground truth of ball motion for an example of simulated, accelerated rotations.

## 278 5. Discussion

279 An optical flow-based method for tracking of spherical treadmill motion  
 280 using a single high-resolution camera was developed and integrated with  
 281 virtual reality projection. Treadmill motion was monitored at 500 Hz, the  
 282 display update rate was 120 Hz, and the total latency for an entire VR cycle  
 283 was  $30 \pm 8$  ms (from reading out ball movement to projecting the updated  
 284 VR frame on the display). The system combines the flexibility of optical flow  
 285 based tracking with the convenience of using a single high resolution camera,  
 286 off-the-shelf components, and open source software.

287 Compared with an optical mouse based system the approach developed



Table 2: Latency time breakdown for the closed-loop virtual reality setup

Component	Time, ms
Camera exposure	0.1
Sensor readout	1.2
Transfer to PC	<2
Tracking algorithm	2
VR rendering + Transfer to DMD over DisplayPort + display latency	$25 \pm 8$
Total	$30 \pm 8$

here offers independent control over the different components of the system, such as camera settings and optical flow processing algorithm. Image processing is therefore not limited to a preset and unknown algorithm, but a suitable approach can be selected and its parameters can be tuned (see Appendix for a comparison of all optical flow algorithms tested). Additionally, the resulting tracking accuracy and the operation range (which for all optical flow based methods depends on the combination of field of view, movement speeds and processing algorithm) can be tested using simulated data. Different from approaches using optical mice, tracking of all three degrees of freedom of ball rotation is achieved with a single camera which can be positioned flexibly and can easily aligned and calibrated. This also facilitates adapting the system to a variety of experimental conditions and scales.

As an alternative to optical flow approaches, a solution relying on a unique patterning of the ball surface was implemented in [6]. While this pattern matching approach avoids integration errors, optical flow can on the other hand track any surface with sufficient speckle contrast under a variety of lighting conditions and can be adapted to different ball sizes.

Compared to cameras, optical mouse sensors typically have a higher frame rate (at the cost of lower pixel resolution) and time jitter and latencies are shorter on a dedicated processing chip. For example, [3] measured the temporal accuracy of a customized optical mouse sensor and found a tracking latency of less than  $500 \mu\text{s}$ , compared to 2 ms here. However, as the latency time breakdown shows (see Table 2), the delay is mostly caused by the standard display interface (DisplayPort). Since the camera tracking latencies and

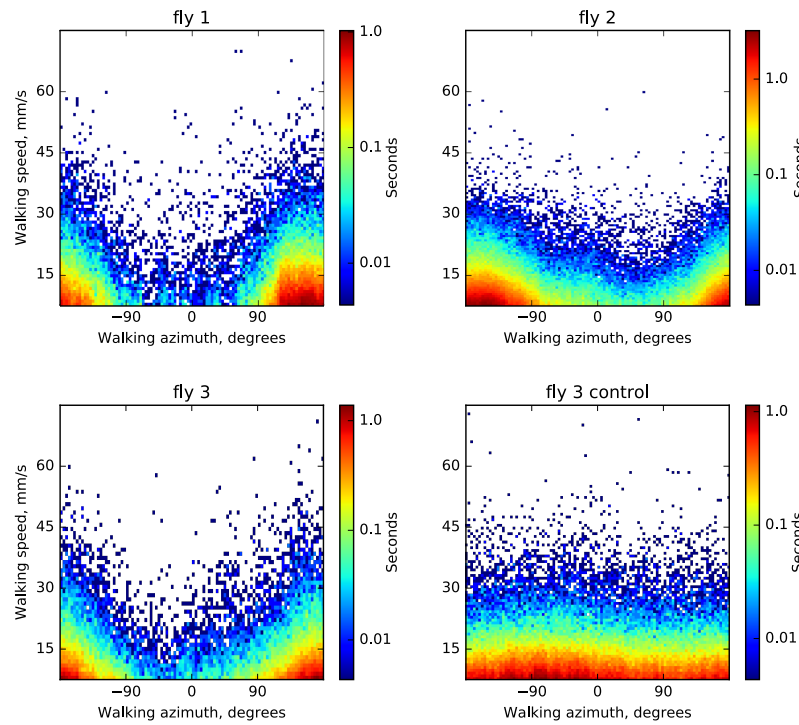


Figure 8: Walking speed and walking direction of three flies walking in closed loop with a bright stripe on a black screen (see text for details). Color indicates histogram bins (binned with a grid of  $100 \times 100$  pixels) of time spent in each state for a total trial length between 15 and 25 minutes (depending on the fly)). In darkness the flies don't show a pronounced orientation preference (as shown for fly 3, control).

312 jitter are small compared to the display latencies and jitter, this should have  
313 a very limited impact on VR behavior. The overall latency of the closed-  
314 loop virtual reality system was  $30 \pm 8$  ms on average, comparable to optical  
315 mice-based solutions and about three times faster than another camera-based  
316 pattern matching approach [6]. The display update rate of typical virtual  
317 reality setups with data transfer through the display port is generally limited  
318 to 60 to 120 Hz. Typical latencies found with such PC and camera-based  
319 solutions are between 50-90 ms [18, 6] and have been shown to be accept-  
320 able for closed-loop behavior experiments with a variety of species [1, 18]. We  
321 tested the system for behavioral performance in fruit flies and found a robust  
322 avoidance response of a bright stripe under the display conditions tested.

323 Overall, a VR system based on optical flow measurements with a high-

324 resolution camera was developed with short latencies suitable for VR exper-  
325 iments at different scales.

## 326 6. Acknowledgements

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331 and Dan Turner-Evans for help with IR illumination.

## 332 7. Appendix

### 333 7.1. Tracking settings

334 Tracking-specific settings are stored in the `Config\tracking.cfg` file and  
335 include the path to the camera configuration file, the name of the camera to  
336 be used for tracking, as well as calibration coefficients as explained in section  
337 3.1. Those parameters can be found in the corresponding sections of the  
338 config file:

```
339 [ camera ]
340 LoadCameraSettings=true
341 CameraSettingsFile=Config\camera_settings.pfs
342 TrackingCameraName=ball_cam
343
344 [ calibration ]
345 BallCenterX=112.0
346 BallCenterY=70.0
347 BallRadius=116.0
348 CxyRad=100.31
349 CxyTan=76.85
350 Cz=20.63
```

351 The camera was set to capture images at 500 fps, and the exposure time  
352 was set to 100  $\mu$ s to prevent motion blurring (and required using sufficient  
353 infrared intensity). The resolution was set to 224 $\times$ 140, the highest resolution  
354 that was handled by our system in the available 2 ms per frame processing  
355 time. The settings of the camera can be saved in the Pylon configuration file  
356 and set to be loaded in the tracking configuration.

## 357 7.2. DMD settings

358 In order to use the LightCrafter 6500 DMDs at a 120 Hz display rate,  
359 they need to be connected through the DisplayPort interface and have to be  
360 configured to use Dual pixel clock mode, run in Video Pattern mode: this  
361 is an operation mode where each frame arriving through the video interface  
362 (RGB 8bpp) is regarded as 24 independent bit planes and each can be inde-  
363 pendently selected to be displayed with an arbitrary timing diagram. In this  
364 mode the most significant bit of the green channel is taken and displayed for  
365 7 ms, resulting in 120 fps display rate. Additionally, it might be necessary to  
366 enable 120 Hz refresh rate in the Windows display adapter settings for both  
367 DMDs, which are visible to the operating system as normal displays.

368 The most convenient way to automate the DMD configuration is by saving  
369 these settings in a batch file (sequence of commands sent to the DMD) and  
370 upload it to the DMD as a startup script (this is a feature supported by  
371 LightCrafter 6500).

## 372 7.3. Virtual environment settings

373 Virtual reality has several configuration parameters which are specified  
374 in the `Config\vr.cfg`. It has game-engine specific parameters in the cor-  
375 responding section, like the position of the VR window in screen coordi-  
376 nates and its size. Since we used two DMDs, it is spanned over twice  
377 the resolution of the DMD in width and located to the right of the main  
378 screen. `VSync` is disabled, since it is otherwise introduces additional la-  
379 tency to the display. `minFps` is set equal to the refresh rate of the displays;  
380 `maxFps` and `maxInactiveFps` are capping the update rate of the game en-  
381 gine. `maxInactiveFps` is the frame rate limit for a window while it is not in  
382 focus, i.e. when the user is working with other applications. This is almost  
383 always the case, since the game window is not visible for the computer user  
384 and other applications, for example for monitoring the VR, are used. The  
385 default value is 60 fps, and therefore needs to be increased in order to use  
386 the full capacity of the 120 Hz DMD displays.

The “transforms” section of the configuration defines how the ball ro-  
tation maps to motion in the virtual environment. The tracking signal is  
calibrated so that it presents angular displacements of the ball (in radians)  
about the x-, y-, and z- axes of the camera. Depending on how the camera  
is oriented relative to forward walking direction of the animal, the mapping  
varies. `ballXYZtoArenaXYZ` is a  $3 \times 3$  rotation matrix, and

$$\text{arena\_displacement} = \text{ballXYZtoArenaXYZ} \cdot \text{ball\_displacement}.$$

Similarly, the turning in the virtual environment is calculated from the ball displacement as a dot product with `ballXYZtoArenaYaw`:

```
arena_yaw_displacement = ballXYZtoArenaYaw · ball_displacement.
```

In the provided example the tracking camera is placed behind the ball and is aligned with the forward walking direction of the fly:

```
[ transforms ]
ballXYZtoArenaXYZ=0 0 -3.0 0 0 0 3.0 0 0
ballXYZtoArenaYaw=0 -57.3248 0
```

The 3 mm radius of the ball is incorporated in the motion transform, and the yaw rotation contains a conversion from radians to degrees.

#### 7.4. *Arena scripts*

Although the Urho3D game engine allows saving and loading 3D environments in XML format, a more readable and convenient way is to declare environment properties in a script. Urho3D supports a compiled scripting language called AngelScript [19], which is object-oriented, and features syntax similar to C++. The scene is represented in the game engine as a hierarchical tree structure: each entity of the 3D environment is a node that can be attached to either other nodes or to the root node of the scene. The coordinates of the nodes are always interpreted in the coordinate frame of their parent nodes. In-detail information is provided in the game engine’s manual [20], and sample arenas are documented. Dithering, which is required for displaying grayscale images with a DMD operating in binary display mode is done with a postprocessing shader, which is added to the rendering pipeline of the game engine. The configuration of the VR display is also shown in a documented example script.

#### 7.5. *Ground truth datasets*

One ground truth dataset was recorded with a 60 mm polystyrene foam ball ( $10 \times$  scale model) mounted on a stepper motor [6]. The motor was controlled using Grbl firmware for Arduino boards [21]. The distance from the camera to the ball was scaled accordingly  $10\times$  as well. Sequences of 1000 frames were recorded with an angular step size of 0.281, 0.562, 1.124 and 1.686 degrees per frame, in two orientations corresponding to y- and z-rotations.

417 A second ground truth dataset consisted of simulated data which allowed  
 418 full control over all parameters: a 3D scene was reconstructed with Blender  
 419 3D including a camera with focal parameters identical to the actual tracking  
 420 setup, a light source and a ball with grainy surface texture. The model of  
 421 the ball was rotated and rendered using Blender’s API and in-built Python  
 422 interpreter. Two simulated rotation sequences were created: The first one  
 423 was used for calibration and accuracy validation and consisted of rotations  
 424 of  $1^\circ$  per frame (corresponding to about half of the maximal walking speed  
 425 of the fly), with 30 different rotation axes and 100 frames for each axis. The  
 426 second dataset consisted of rotations of exponentially increasing magnitude  
 427 from  $0^\circ$  to  $2^\circ$  per frame, intended to test the operating range of the tracking  
 428 algorithm. The orientations of the rotational axes for both datasets were  
 429 selected evenly distributed.

430 An actual tracking camera image is shown side-to-side with a simulated  
 431 image in Figures 9 and 10: the real image has out-of-focus areas, some bright  
 432 speckles from direct reflection of the lighting, and an overall uneven bright-  
 433 ness distribution.

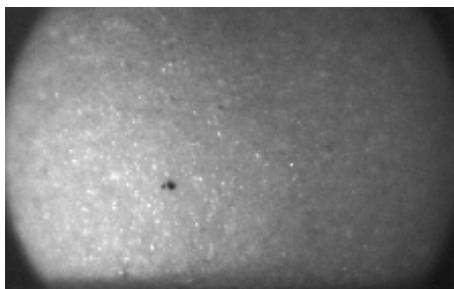


Figure 9: Frame captured by tracking camera in VR setup.

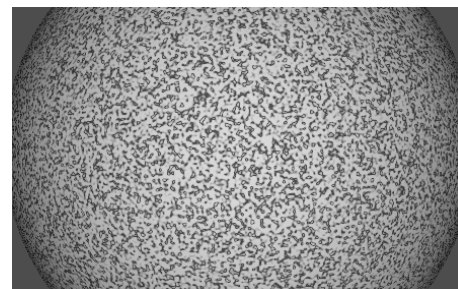


Figure 10: Frame from simulated ball rotation sequence.

#### 434 7.6. Optical flow algorithms

435 Algorithms for computing optical flow vary significantly in computational  
 436 complexity, run-time and resulting accuracy. OpenCV library features sev-  
 437 eral implementations of the optical flow estimation algorithms (as of version  
 438 3.4.1) and the algorithms tested here were: Lucas-Kanade sparse feature  
 439 tracking [22], Gunnar Farneback’s algorithm of dense optical flow [23], the  
 440 optical flow algorithm by Brox et. al. [24], Dual TV- $L_1$  [25], Dense Inverse

441 Search (DIS) [26], SimpleFlow [27], DeepFlow [28] and PCAFlow [29] (see  
442 Table 3).

443 The speedup through reducing the size of the ROI varies depending on the  
444 algorithm, which motivated a comparison of the run-times depending on the  
445 input size (Fig 12). To achieve a tracking frequency of 500 frames per second,  
446 the run-time should not exceed 2 ms, preferably be less to accommodate the  
447 frame preprocessing and ball motion estimation calculations. The algorithms  
448 relying on sparse features pre-selection fail in smaller ROIs (Sparse-to-dense,  
449 PCAFlow). Farneback, although being slow with default parameters, could  
450 be sped up significantly by reducing the number of scale levels and by limiting  
451 the internal iterations. It turned out to be the algorithm most suitable for  
452 the application, since it can handle a ROI of up to  $60 \times 140$  px in under 2  
453 ms when tuned for speed.

Table 3: Tracking performance on the the simulated rotation dataset, using full frame optical flow.

Algorithm	Mean magnitude error, %	Mean orientation error, degrees	Time, ms
Dual TV-L1 (CUDA)	4.9	1.95	953.2
DeepFlow	1.7	0.76	162.4
Brox (CUDA)	3.9	1.69	70.4
PCAFlow (10x10 priors)	1.6	0.73	33.1
PCAFlow (default priors)	2.0	0.84	26.4
Farneback	1.2	0.54	14.6
Lucas-Kanade (CUDA)	1.9	0.83	13.2
Farneback (CUDA)	1.2	0.54	10.5
SparseToDense	2.0	0.83	8.2
DIS (medium)	3.6	1.6	4.6
DIS (fast)	14.5	6.48	1.9
DIS (ultrafast)	18.7	8.29	0.5

### 454 7.7. Selecting ROI parameters

455 The ROI is a small part of the full frame used to identify the rotation  
456 direction of the ball from optical flow. The ROI is limited by two concentric

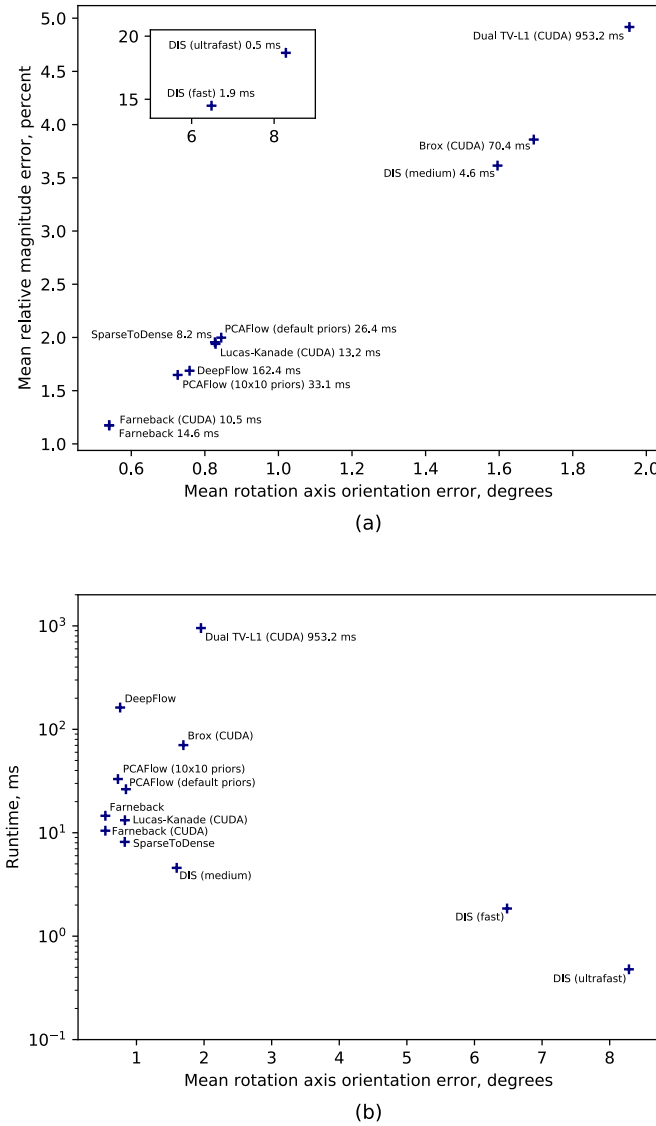


Figure 11: Ball rotation estimation errors (a) and run-time of the optical flow methods on full frame of the simulated dataset

457 rings with their centers coinciding with the center of the frame, which is also  
 458 selected to agree with the center of the tracked ball. By specifying the radius  
 459 of the ROI  $\rho_{ROI}$  (px) and its width  $\Delta\rho_{ROI}$  (px), one can vary the run-times



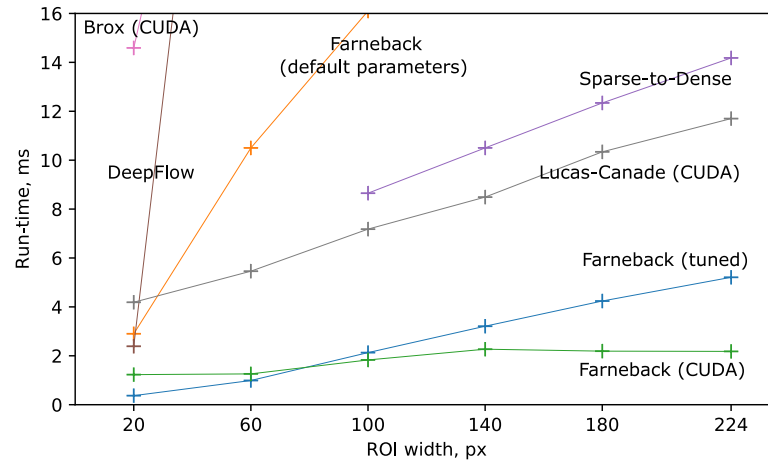


Figure 12: Run-time of optical flow algorithms depending on input size (ROI width  $\times$  140 px). Measured as average over 3000 frames with simulated dataset with accelerating rotations.

of the fitting algorithm and its accuracy. The ROI parameters are chosen by running the tracking algorithm on simulated datasets (constant  $1^\circ$  per frame angular velocity) and finding the variance in the estimated angular velocity magnitude and orientation compared to ground truth. Figure 13 shows the summed magnitude and orientation error (normalized) depending on  $\rho_{ROI}$ , using  $\Delta\rho_{ROI} = 10$  px. The lowest value was achieved for  $\rho_{ROI} = 0.27$ ,  $\rho_{max} = 60$  px.

### 7.8. Calibration coefficients

The coefficients  $c_{xy \text{ rad}}$ ,  $c_{xy \text{ tan}}$  and  $c_z$  in expression (8) are required for relating the optical flow in pixels (as determined with the optical flow algorithm) to the actual ball displacement in radians. Apart from a unit conversion factor these coefficients depend on the camera magnification, the size of the ball, and the position of the ROI on the ball. These factors can be determined in any of the following ways: by correlating optical flow induced by known ball rotations (with a ground truth dataset); similar, but using modelled optical flow with the pinhole camera model as in expressions (5-7, provided the optical parameters of the camera are known); or using a second synchronized camera. These methods are described in more detail in the

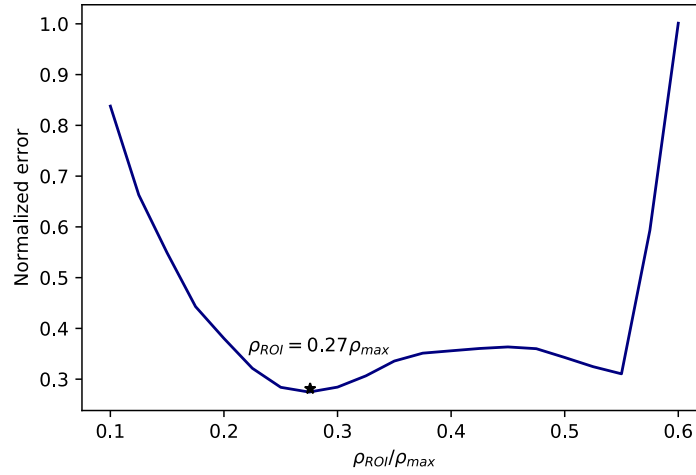


Figure 13: Summed tracking orientation and rotation errors (normalized), obtained by running the tracking algorithm with  $\Delta\rho_{ROI} = 10$  px.  $\rho_{max}$  is the visual radius of the ball (116 px).

478 following subsections.

### 479 7.8.1. Calibration with ground truth dataset

480 The optical flow in the tracking ROI was calculated on simulated data and  
481 its distributions are fitted with function (8). Then the calibration constants  
482 can be found as the ratios:

$$\begin{aligned} c_z &= \frac{\omega_{zGroundTruth}}{\omega_{zFitted}} \\ c_{xy \text{ rad}} &= \frac{\omega_{xyGroundTruth}}{\omega_{xy \text{ radFitted}}} \\ c_{xy \text{ tan}} &= \frac{\omega_{xyGroundTruth}}{\omega_{xy \text{ tanFitted}}}, \end{aligned} \quad (9)$$

483 where  $\omega_{xyGroundTruth}, \omega_{zGroundTruth}$  are the known rotations, and  $\omega_{xy \text{ radFitted}},$   
484  $\omega_{xy \text{ tanFitted}}$  are the results of fitting radial and tangential optical flow distri-  
485 butions in the ROI assuming  $c_{xy \text{ rad}} = c_{xy \text{ tan}} = c_z = 1.0$ .

Using the rendered ball rotation dataset with constant rotations, the fol-

lowing calibration factors were found:

$$c_{xy \text{ rad}} = 100.31 \text{ px/rad} = 1.75 \text{ px}/^\circ,$$

$$c_{xy \text{ tan}} = 76.85 \text{ px/rad} = 1.34 \text{ px}/^\circ,$$

$$c_z = 20.63 \text{ px/rad} = 0.36 \text{ px}/^\circ.$$

#### 486 7.8.2. Calculation of the calibration factors from the scene model

487 Using the ball projection model (equations 3 and 4) with substituted cam-  
488 era and scene parameters, optical flow distribution in the tracking ROI can  
489 be predicted and used for calibration similar to the previous method. With  
490 the help of a scaled-up calibration setup, the camera and scene parameters  
491 were measured as listed in Table 4.

Table 4: Measured parameters of the tracking setup scale model

Parameter	Value
Distance to the ball	1400 mm
Ball radius	30 mm
Image resolution	$224 \times 140$ px
Ball image radius	116 px

492 Using these measurements, and the pinhole camera model (4) the focal  
493 length of the camera can be calculated (assuming the image projection center  
494 is exactly in the middle of the frame,  $center_x = 112$  px,  $center_y = 70$  px):

$$116 \text{ px} = f \frac{30 \text{ mm}}{1400 \text{ mm}}$$

$$f = 116 \text{ px} \frac{1400 \text{ mm}}{30 \text{ mm}} = 5413 \text{ px}$$

$$f_x = f_y = 5410 \text{ px}$$

$$center_x = 112 \text{ px}$$

$$center_y = 70 \text{ px}$$

495 The model with these parameters was used to predict the optical flow  
496 induced by different rotations according to expressions ((5) - (7)).

497 The calibration factors found with this method were close to ones obtained  
498 with ground truth rotations:

$$\begin{aligned} c_{xy \text{ rad}} &= 95.37 \text{ px/rad} = 1.66 \text{ px}/^\circ, \\ c_{xy \text{ tan}} &= 70.31 \text{ px/rad} = 1.21 \text{ px}/^\circ, \\ c_z &= 22.28 \text{ px/rad} = 0.39 \text{ px}/^\circ. \end{aligned}$$

### 499 7.8.3. Two-camera calibration

500 According to the mathematical model described above the z-component  
501 of the angular velocity measured by the tracking camera is detected as the  
502 xy-plane component by a second camera filming the same rotation from an  
503 orthogonal direction (see Figure 14).

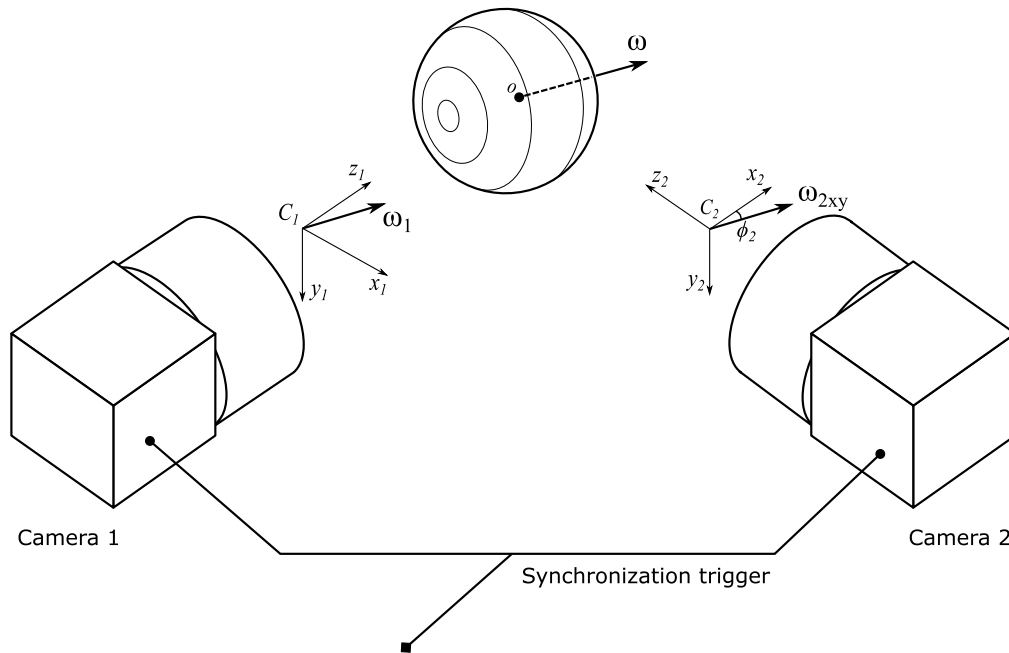


Figure 14: Calibration procedure using two cameras. The z-component of angular velocity of the ball in the reference frame of camera 1 is projected onto the x-axis in the reference frame of camera 2. Calibration is performed by correlating those components to find  $c_{xy}$ .

504 As shown using the optical flow model (Figure 4), a rotation of the ball  
505 around the camera's z-axis results in all of its points moving along circular

trajectories in the frame plane. This means that the angular displacement of the ball equals the angular displacement of the points in the frame along their trajectories, and the tangential optical flow in the ROI induced by this motion is proportional to the radius of the ROI, meaning that  $c_z$  can be estimated with a single camera without measuring any additional parameters.

Different from z-factors,  $c_{xy \text{ rad}}$  and  $c_{xy \text{ tan}}$  depend on the camera properties and the scene geometry. Therefore, a second camera is introduced, filming the ball from an orthogonal direction, as shown in Figure (14). The z-component measured by camera 1 is then equal to the x-component measured by camera 2, and camera 1 can therefore be used to calibrate xy-tracking by camera 2:

$$\begin{cases} c_{xy \text{ rad}} = \frac{c_{z \text{ estimated}} \omega_{z1}}{\omega_{xy \text{ rad}2} \cos \phi_2} \\ c_{xy \text{ tan}} = \frac{c_{z \text{ estimated}} \omega_{z1}}{\omega_{xy \text{ tan}2} \cos \phi_2}, \end{cases} \quad (10)$$

where  $c_{z \text{ estimated}}$  is estimated as described,  $\omega_{z1}$  is measured by camera 1,  $\omega_{xy \text{ rad}2}$ ,  $\omega_{xy \text{ tan}2}$  and  $\phi_2$  are found by fitting function (8) on radial and tangential optical flow in the ROI from camera 2, respectively, initially assuming  $c_{xy \text{ rad}} = c_{xy \text{ tan}} = 1.0$ .  $\phi_2$  is the orientation of the ball's axis of rotation in the xy-plane of camera 2, and the cosine yields the x-axis projection of that rotation, corresponding to measured z-rotation by camera 1.

To perform this calibration, both cameras need to be set up with identical objectives and aligned at the same distance from the tracked ball at right angles (see Figure 14); the ball is let to spin freely (at a speed within the range where the tracking algorithm is valid) with a stream of air while the cameras capture frames synchronized by an external triggering signal.

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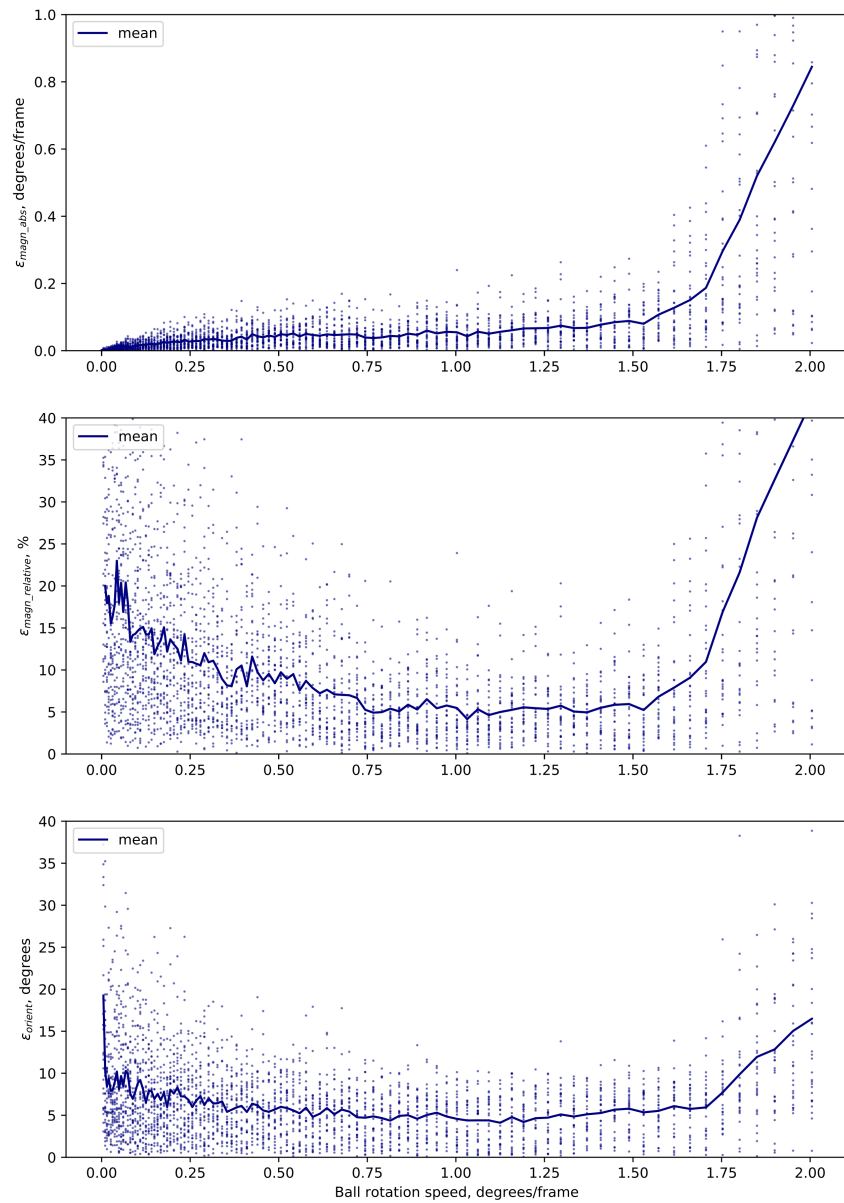


Figure 15: Tracking errors for an example of simulated, accelerated rotations, corresponding to the data shown in Figure 7.